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PRODUCTION INVENTORY MODEL WITH DETERIORATION AND PARTIAL SHORTAGES

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ABSTRACT
Many researchers have conducted extensive studies on the deteriorating inventory strategy. Compared with normal inventory models, besides demand, deteriorating rate is another key factor that impacts significantly on an inventory management system. This paper will mainly concentrate on demand and deteriorating and partial shortages and other factors, such as price discount, inflation, and so on, will also be taken into consideration
KEY WORDS: Inventory, Deteriorating

INTRODUCTION:
According to the study of Wee HM in 1993, deteriorating items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers, film and so on; the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods, and so on. Both of the two categories have the characteristic of short life cycle. For the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items will change and then lose useable value and economic value; for the second category, the items have a short market life cycle. After a period of popularity in the market, the items lose the original economic value due to the changes in consumer preference, product upgrading and other reasons.

ASSUMPTIONS AND NOTATIONS:
The production inventory model for deteriorating items is developed on the basis of the following assumptions and notations.

- I (t) is the inventory level at any time $t \geq 0$.
- The demand rate $R(t) = Ae^{bt}$, where A and b are the positive constants
- The replenishment rate is $K = \gamma R(t)$, where $\gamma > 1$ is a constant.
- A fraction $\theta(t) = \alpha\beta t^{\beta-1}$, $0 < \alpha << 1$, $t > 0$, $\beta \geq 1$ of the on hand inventory deteriorates per unit time.
- The deteriorating units can be neither replaced nor repaired during the cycle time.
- The lead-time is zero.
- C', C_H, C_S, C_D, C_L denote the set up cost for each replenishment, inventory carrying cost per unit time, shortage cost for backlogged items, deterioration cost per unit, the unit cost of lost sales respectively. All of the cost parameters are positive constants.
- Shortages are allowed and backlogging rate is taken as δ, where δ is the backlogging parameter and $0 < \delta <$
- T is the time horizon.
- A single item is considered over the prescribed period of time.

MATHEMATICAL MODEL AND ANALYSIS
Initially the inventory level is zero. The production starts at time t=0 and after t_1, units of time, it reaches to maximum inventory level. After this production stopped and at time $t = t_2$ the inventory level becomes zero. At this time shortage starts developing at time $t = t_3$, it reaches to maximum shortages level. At this

fresh production starts to clear the backlog by the time t = T. Our aim is to find the optimum values of t_1, t_2, t_3, T that minimize. The total average cost C over time horizon (0, T).

Let I(t) be the inventory level at any time t, $0 \le t \le T$.

The governing differential equations of an inventory system in the interval [0, T) are

$$\frac{dI(t)}{dt} + \theta(t)I(t) = K - R(t), \qquad 0 \le t \le t_1 \qquad \text{.....(1)}$$

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -R(t), \qquad t_1 \le t \le t_2 \qquad \text{.....(2)}$$

$$\frac{dI(t)}{dt} = -\delta R(t), \qquad t_2 \le t \le t_3 \qquad \text{.....(3)}$$

$$\frac{dI(t)}{dt} = K - \delta R(t), \qquad t_3 \le t \le T \qquad \text{.....(4)}$$

with the condition, $I(0) = I(t_2) = I(T) = 0$. Using the value of R(t), θ(t) and K the equations (1), (2), (3) and (4) can be written as

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1}I(t) = (\gamma - 1)Ae^{bt}, \qquad 0 \le t \le t_1 \qquad \text{.....(5)}$$

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1}I(t) = -Ae^{bt}, \qquad t_1 \le t \le t_2 \qquad \text{.....(6)}$$

$$\frac{dI(t)}{dt} = -\delta Ae^{bt}, \qquad t_2 \le t \le t_3 \qquad \text{.....(7)}$$

$$\frac{dI(t)}{dt} = (\gamma - \delta)Ae^{bt}, \qquad t_3 \le t \le T \qquad \text{.....(8)}$$

The solution of equation (5) is given by

$$I(t).e^{\alpha t^{\beta}} = A(\gamma - 1)\int e^{\alpha t^{\beta}} e^{bt} dt + K_1$$

where K_1 is the constant of integration

$$I(t).e^{\alpha t^{\beta}} = A(\gamma - 1)\int (1 + bt + \alpha t^{\beta} + b\alpha t^{\beta+1})dt + K_1$$

$$I(t) = K_1 e^{-\alpha t^{\beta}} + A(\gamma - 1)\left[t + \frac{bt^2}{2} + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{\beta+2}\right]e^{-\alpha t^{\beta}}$$

$$I(t) = K_1(1 - \alpha t^{\beta}) + A(\gamma - 1)\left[t + \frac{bt^2}{2} - \frac{\alpha\beta t^{\beta+1}}{\beta+1} - \frac{b\alpha\beta t^{\beta+2}}{2(\beta+2)}\right]$$

But I(0) = 0, therefore $K_1 = 0$

$$I(t) = A(\gamma - 1)\left[t + \frac{bt^2}{2} - \frac{\alpha\beta t^{\beta+1}}{\beta+1} - \frac{b\alpha\beta t^{\beta+2}}{2(\beta+2)}\right], \; 0 \le t \le t_1 \qquad \text{.....(9)}$$

The solution of equation (6) is given by

$$I(t).e^{\alpha t^{\beta}} = -A\int e^{\alpha t^{\beta}} e^{bt} dt + K_2$$

where K_2 is the constant of integration

$$I(t) = K_2 e^{-\alpha t^{\beta}} - A\left[t + \frac{bt^2}{2} + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{\beta+2}\right] e^{-\alpha t^{\beta}}$$

But $I(t_2) = 0$ therefore .

$$K_2 = A\left[t_2 + \frac{bt_2^2}{2} + \frac{\alpha t_2^{\beta+1}}{\beta+1} + \frac{b\alpha t_2^{\beta+2}}{\beta+2}\right]$$

$$I(t) = A\left[t_2 + \frac{bt_2^2}{2} + \frac{\alpha t_2^{\beta+1}}{\beta+1} + \frac{b\alpha t_2^{\beta+2}}{\beta+2}\right] e^{-\alpha t^{\beta}} - A\left[t + \frac{bt^2}{2} + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{\beta+2}\right] e^{-\alpha t^{\beta}}$$

$$I(t) = A\left[(t_2 - t) + \frac{b}{2}(t_2^2 - t^2) + \frac{\alpha\beta t^{\beta+1}}{(\beta+1)} + \frac{b\alpha\beta t^{\beta+2}}{2(\beta+2)} - \alpha t_2 t^{\beta} - \frac{b\alpha t_2^2 t^{\beta}}{2} + \frac{\alpha t_2^{\beta+1}}{\beta+1} + \frac{b\alpha t_2^{\beta+2}}{\beta+2}\right], t_1 \le t \le t_2$$

.....(10)

The solution of equation (7) is given by

$$I(t) = -\delta A\frac{e^{bt}}{b} + K_3$$

where K_3 is the constant of integration

Total number of unit holding is given by

$$I_H = \int_0^{t_1} I(t)dt + \int_{t_1}^{t_1} I(t)dt$$

$$= A(\gamma-1)\left[\frac{t^2}{2} + \frac{bt^3}{6} - \frac{\alpha\beta t^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{b\alpha\beta t^{\beta+3}}{2(\beta+2)(\beta+3)}\right]_0^{t_1} + A\left[t_2 t - \frac{t^2}{2}\right.$$

$$+ \frac{b}{2}\left(t_2^2 t - \frac{t^3}{3}\right) + \frac{\alpha\beta t^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\alpha\beta t^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha t_2 t^{\beta+1}}{\beta+1} - \frac{b\alpha t_2^2 t^{\beta+1}}{2(\beta+1)}$$

$$\left. + \frac{\alpha t_2^{\beta+1} t}{\beta+1} + \frac{b\alpha t_2^{\beta+2} t}{\beta+2}\right]_{t_1}^{t_2}$$

$$I_H = A\left[(\gamma-1)\left\{\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{b\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}\right\} + t_2^2 - \frac{t_2^2}{2} + \frac{b}{2}\left(t_2^3 - \frac{t_2^3}{3}\right)\right.$$

$$+ \frac{\alpha\beta t_2^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\alpha\beta t_2^{\beta+3}}{2(\beta+2)(\beta+3)} - \frac{\alpha t_2^{\beta+2}}{\beta+1} - \frac{b\alpha t_2^{\beta+3}}{2(\beta+1)} + \frac{\alpha t_2^{\beta+2}}{\beta+1} + \frac{b\alpha t_2^{\beta+3}}{\beta+2} - t_2 t_1$$

$$+\frac{t_1^2}{2}-\frac{b}{2}\left(t_2^2t_1-\frac{t_1^3}{3}\right)-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{b\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha t_2 t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_2^2 t_1^{\beta+1}}{2(\beta+1)}$$

$$\left.-\frac{\alpha t_2^{\beta+1}t_1}{\beta+1}-\frac{b\alpha t_2^{\beta+2}t_1}{(\beta+2)}\right]$$

$$I_H=A\left[\gamma\left\{\frac{t_1^2}{2}+\frac{bt_1^3}{6}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{b\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}\right\}+\frac{t_2^2}{2}+\frac{bt_2^3}{3}+\frac{\alpha\beta t_2^{\beta+2}}{(\beta+1)(\beta+2)}\right.$$

$$+\frac{b\alpha\beta t_2^{\beta+3}}{(\beta+1)(\beta+3)}-t_2t_1-\frac{bt_2^2t_1}{2}+\frac{\alpha t_2 t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_2^2 t_1^{\beta+1}}{2(\beta+1)}-\frac{\alpha t_2^{\beta+1}t_1}{\beta+1}$$

$$\left.-\frac{b\alpha t_2^{\beta+2}t_1}{\beta+2}\right] \qquad .....(11)$$

Total Amount of deteriorated units is given by

$$I_D=\int_0^{t_1}\theta(t)I(t)dt+\int_{t_1}^{t_2}\theta(t)I(t)dt$$

$$I_D=A\alpha\beta\left[(\gamma-1)\int_0^{t_1}\left(t^\beta+\frac{bt^{\beta+1}}{2}\right)dt+\int_{t_1}^{t_2}(t_2t^{\beta-1}-t^\beta+\frac{b}{2}t_2^2t^{\beta-1}-\frac{b}{2}t^{\beta+1})dt\right]$$

$$=A\alpha\beta\left[(\gamma-1)\left(\frac{t^{\beta+1}}{\beta+1}+\frac{bt^{\beta+2}}{2(\beta+2)}\right)_0^{t_1}+\left(\frac{t_2t^\beta}{\beta}-\frac{t^{\beta+1}}{\beta+1}+\frac{bt_2^2t^\beta}{2\beta}-\frac{bt^{\beta+2}}{2(\beta+2)}\right)_{t_1}^{t_2}\right]$$

$$=A\alpha\beta\left[(\gamma-1)\left(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)}\right)+\frac{t_2^{\beta+1}}{\beta}-\frac{t_2^{\beta+1}}{\beta+1}+\frac{bt_2^{\beta+2}}{2\beta}-\frac{bt_2^{\beta+2}}{2(\beta+2)}\right.$$

$$\left.-\frac{t_2t_1^\beta}{\beta}+\frac{t_1^{\beta+1}}{\beta+1}-\frac{bt_2^2t_1^\beta}{2\beta}+\frac{bt_1^{\beta+2}}{2(\beta+2)}\right]$$

$$I_D=A\alpha\beta\left[\gamma\left(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)}\right)+\frac{t_2^{\beta+1}}{\beta(\beta+1)}+\frac{bt_2^{\beta+2}}{\beta(\beta+2)}-\frac{t_2t_1^\beta}{\beta}-\frac{bt_2^2t_1^\beta}{2\beta}\right] \qquad .....(12)$$

Total number of shortage units is given by

$$I_S=-\int_{t_2}^{t_3}I(t)dt-\int_{t_3}^{T}I(t)dt$$

$$=-\left[\int_{t_2}^{t_3}\frac{\delta A}{b}\left(e^{bt_2}-e^{bt}\right)dt+\int_{t_3}^{T}\frac{A(\gamma-\delta)}{b}\left(e^{bt}-e^{bT}\right)dt\right]$$

$$I_S=-\left[\frac{\delta A}{b}\left\{(t_3-t_2)e^{bt_2}+\frac{e^{bt_2}-e^{bt_3}}{b}\right\}+\frac{A(\gamma-\delta)}{b}\left\{\frac{e^{bT}-e^{bt_3}}{b}+(t_3-T)e^{bT}\right\}\right] \qquad .....(13)$$

Total amount of lost sales is given by

$$I_L=\int_{t_2}^{t_3}(1-\delta)R(t)dt+\int_{t_3}^{T}(1-\delta)R(t)dt$$

$$= \int_{t_2}^{T} (1-\delta)R(t)dt$$

$$= \int_{t_2}^{T} (1-\delta)Ae^{bt}dt$$

$$I_L = \frac{(1-\delta)A}{b}\left[e^{bT} - e^{bt_2}\right] \qquad .....(14)$$

from equation (9) we have

$$I(t_1) = A(\gamma-1)\left[t_1 + \frac{bt_1^2}{2} - \frac{\alpha\beta t_1^{\beta+1}}{\beta+1} - \frac{b\alpha\beta t_1^{\beta+2}}{2(\beta+2)}\right] \qquad .....(15)$$

from equation (10) we have

$$I(t_1) = A\left[(t_2 - t_1) + \frac{b}{2}(t_2^2 - t_1^2) + \frac{\alpha\beta t_1^{\beta+1}}{\beta+1} + \frac{b\alpha\beta t_1^{\beta+2}}{2(\beta+2)} - \alpha t_2 t_1^{\beta}\right.$$

$$\left. - \frac{b\alpha t_2^2 t_1^{\beta}}{2} + \frac{\alpha t_2^{\beta+1}}{\beta+1} + \frac{b\alpha t_2^{\beta+2}}{\beta+2}\right] \qquad .....(16)$$

By equations (17) and (18), we get

$$\gamma\left[t_1 + \frac{bt_1^2}{2} - \frac{\alpha\beta t_1^{\beta+1}}{\beta+1} - \frac{b\alpha\beta t_1^{\beta+2}}{2(\beta+2)}\right] = \left[t_2 + \frac{bt_2^2}{2} - \alpha t_2 t_1^{\beta} - \frac{b\alpha t_2^2 t_1^{\beta}}{2} + \frac{\alpha t_2^{\beta+1}}{\beta+1} + \frac{b\alpha t_2^{\beta+2}}{\beta+2}\right]$$

Now we consider as,

$$t_2 = f(t_1) \qquad .....(17)$$

from equations (11), we have

$$I(t_3) = \frac{\delta A}{b}\left[e^{bt_2} - e^{bt_3}\right] \qquad .....(18)$$

from equation (12), we have

$$I(t_3) = \frac{A(\gamma-\delta)}{b}\left[e^{bt_3} - e^{bT}\right] \qquad .....(19)$$

On comparing equations (20) and (21), we have

$$\gamma e^{bt_3} = \delta e^{bt_2} + (\gamma-\delta)e^{bT}$$

Now we are consider as ,

$$t_3 = g(T, t_1) \qquad .....(20)$$

Hence, the total average cost of the system per unit time is given by

$$C = \frac{1}{T}\left[C' + C_H I_H + C_D I_D + C_S I_S + C_L I_L\right] = \frac{R}{T} \qquad ....(21)$$

$$C = \frac{1}{T}\left[C' + C_H A\left\{\gamma\left(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{b\alpha\beta t_1^{\beta+2}}{2(\beta+2)(\beta+3)}\right) + \frac{[f(t_1)]^2}{2} + \frac{b[f(t_1)]^3}{3}\right.\right.$$

$$+\frac{\alpha\beta[f(t_1)]^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b\alpha\beta[f(t_1)]^{\beta+3}}{(\beta+1)(\beta+3)}-t_1f(t_1)-\frac{bt_1[f(t_1)]^2}{2}+\frac{\alpha t_1^{\beta+1}f(t_1)}{\beta+1}+\frac{b\alpha t_1^{\beta+1}[f(t_1)]^2}{2(\beta+1)}$$

$$-\frac{\alpha t_1[f(t_1)]^{\beta+1}}{\beta+1}-\frac{b\alpha t_1[f(t_1)]^{\beta+2}}{\beta+2}\Bigg\}+C_DA\alpha\beta\Bigg\{\gamma\left(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)}\right)+\frac{[f(t_1)]^{\beta+1}}{\beta(\beta+1)}+\frac{b[f(t_1)]^{\beta+2}}{\beta(\beta+2)}$$

$$-\frac{t_1^{\beta}[f(t_1)]}{\beta}-\frac{bt_1^{\beta}[f(t_1)]^2}{2\beta}\Bigg\}-C_S\Bigg\{\frac{\delta A}{b}\left(\left(g(T,t_1)-f(t_1)\right)e^{bf(t_1)}+\frac{e^{bf(t_1)}-e^{bg(T,t_1)}}{b}\right)$$

$$+\frac{A(\gamma-\delta)}{b}\left(\frac{e^{bT}-e^{bg(T,t_1)}}{b}+(g(T,t_1)-T)e^{bT}\right)\Bigg\}+C_L\frac{(1-\delta)A}{b}\left\{e^{bT}-e^{bf(t_1)}\right\}\Bigg]=\frac{R}{T}.(22)$$

CONCLUSION

The demand of seasonable and fashionable products can be described well with this function, as the nature of demand of these products is increasing at the beginning of the season, steady in the mid of the season, and decreasing at the end of the season. To the best of our knowledge this demand pattern studied for the first time, at least, using so general functions for the non steady periods.

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